

64. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude  $f = 4400$  N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is  $\Delta E_{\text{th}} = fd$  (Eq. 8-29) where  $d = 3.7$  m in part (a) (but will be replaced by  $x$ , the spring compression, in part (b)).

- (a) With  $W = 0$  and the reference level for computing  $U = mgy$  set at the top of the (relaxed) spring, Eq. 8-31 leads to

$$U_i = K + \Delta E_{\text{th}} \implies v = \sqrt{2d \left( g - \frac{f}{m} \right)}$$

which yields  $v = 7.4$  m/s for  $m = 1800$  kg.

- (b) We again utilize Eq. 8-31 (with  $W = 0$ ), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing  $U = mgy$  as we did in part (a), we end up with gravitational potential energy equal to  $mg(-x)$  at that bottom-most point, where the spring (with spring constant  $k = 1.5 \times 10^5$  N/m) is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

where  $K = \frac{1}{2}mv^2 = 4.9 \times 10^4$  J using the speed found in part (a). Using the abbreviation  $\xi = mg - f = 1.3 \times 10^4$  N, the quadratic formula yields

$$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90 \text{ m}$$

where we have taken the positive root.

- (c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance  $d'$  above the relaxed position of the spring). We assume  $d' > x$ . We now use the bottom-most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2 = mgd' + fd' \implies d' = \frac{kx^2}{2(mg + d)} = 2.8 \text{ m} .$$

- (d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions – but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount  $d_{\text{eq}}$  given by

$$mg = kd_{\text{eq}} \implies d_{\text{eq}} = \frac{mg}{k} = 0.12 \text{ m} .$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original  $U = mgy$  becomes  $mg(d_{\text{eq}} + d)$ . In that final position, then, the gravitational energy is zero and the spring energy is  $\frac{1}{2}kd_{\text{eq}}^2$ . Thus, Eq. 8-31 becomes

$$\begin{aligned} mg(d_{\text{eq}} + d) &= \frac{1}{2}kd_{\text{eq}}^2 + fd_{\text{total}} \\ (1800)(9.8)(0.12 + 3.7) &= \frac{1}{2}(1.5 \times 10^5)(0.12)^2 + (4400)d_{\text{total}} \end{aligned}$$

which yields  $d_{\text{total}} = 15$  m.